Optimization of k-Space Trajectories by Bayesian Experimental Design

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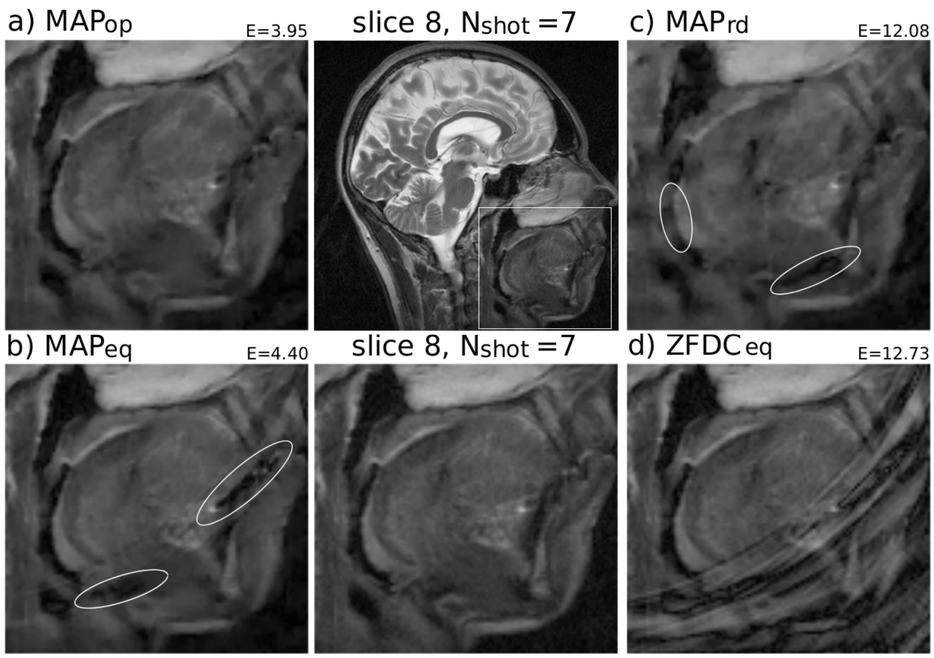
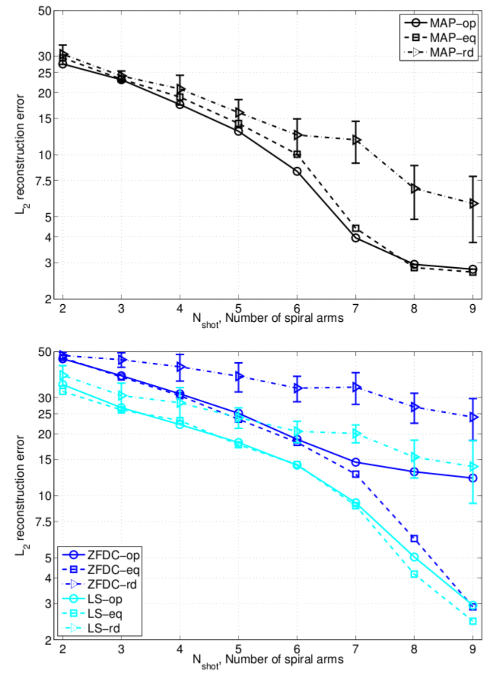
Introduction MR image reconstruction from undersampled *k*-space can be improved by nonlinear denoising estimators since they incorporate statistical prior knowledge about image sparsity [1]. Reconstruction quality depends crucially on the undersampling design (*k*-space trajectory), in a manner complicated by the nonlinear and signal-dependent characteristics of these methods. Due to the nonlinearity, it is not obvious how to undersample best such that sparse reconstruction is as faithful as possible. We propose an algorithm to assess and optimize *k*-space trajectories for sparse MRI reconstruction, based on Bayesian experimental design, which is scaled up to full MR images by a novel variational relaxation to iteratively reweighted FFT or gridding computations. Designs are built sequentially by adding phase encodes predicted to be most informative, given the combination of previous measurements and image prior information.





Methods We consider spiral trajectories, composed of N_{shot} Archimedian interleaves leaving the *k*-space origin at offset angles θ_0, where non-Cartesian measurements are re-gridded from data acquired on a Siemens 3T scanner (16 sagittal slices through brain of healthy volunteer; TSE, 23 echos/exc., 120° refocusing pulses, 1x1x4mm³, 256x256 pixels; low-frequency phase correction from 32x32 central matrix). We compare half-Fourier designs ($\theta_0 \in [0,\pi)$), reconstructions are scored by L2-distance to image from complete data. Different reconstruction methods (ZFDC: linear zero-filling, density compensation; LS: linear least squares; MAP: nonlinear sparse estimation [1]) are compared under design choices for θ_0 (eq: equispaced in [0,π); rd: drawn uniformly at random; op: optimized by our Bayesian algorithm). A Nyquist spiral ($\theta_0 \in [0,2\pi)$) has N_{shot}=16.

Results Left: Close-ups, N_{shot}=7 interleaves. (a) MAPop (optimized, our method): Best results (center: true image). (b) MAPeq (equispaced): Aliasing artifacts due to regular sampling. (c) MAPrd (random): Reduced apparent resolution. (a-c) use sparse reconstruction [1]. (d): ZFDCeq (linear reconstruction): Loss of features due to strong artifacts (compare to MAPeq) Right, top: Reconstruction error vs. N_{shot}, sparse estimation [1]. Optimized designs improve upon equispaced for N_{shot} <8 (sub-Nyquist for strictly real-valued signal) by up to 16%. Randomly drawn designs perform badly. Right, bottom: Same for linear LS/ZFDC reconstruction. Larger errors in general. Design optimization less effective (specialized to sparse reconstruction).

Conclusions Nonlinear estimators tend to reconstruct better images from undersampled data than linear techniques, but success depends strongly on the *k*-space sampling design. Trajectories optimized by our algorithm lead to less artifacts than equispaced designs at the same cost; both strongly improve upon randomly drawn designs. In our Bayesian framework, further knowledge can be specified through modifications of the forward model and signal prior distribution. Our algorithm iterates over primitives for FFT or gridding, and can be implemented easily based on code for these.

[1] M. Lustig, D. Donoho, and J. Pauly. Sparse MRI: The application of compressed sensing for rapid MR imaging. Magn. Reson. Med., 85(6):1182–1195, 2007.