Learning optimal EEG features across time, frequency and space.

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NIPS06 Workshop on Trends in BCI
Outline

Motivation
Source types in EEG based BCI

Automatic Feature Selection
Learning Spatial Features
Feature selection as Model Selection
Spectral/Temporal Filtering

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Learning optimal EEG features across time, frequency and space.
The current approach to learning in BCIs

Current BCI use learning in two distinct phases,

1. **Feature Extraction** – where we attempt to extract features which lead to good classifier performance,

2. **Classification** – usually a simple linear classifier, (SVM, LDA, Gaussian), because

   “Once we have good features the classifier doesn’t really matter”
The current approach to learning in BCIs

Current BCI use learning in two distinct phases,

1. **Feature Extraction** – where we attempt to extract features which lead to good classifier performance, using,
   - prior-knowledge, the 7-30Hz band for ERDs
   - maximising r-scores,
   - maximising ’independence’ (ICA)
   - maximising the ratios of the class variances (CSP)

2. **Classification** – usually a simple linear classifier, (SVM, LDA, Gaussian), because
   
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This seems wrong!

Note
The objectives used in feature extraction are not good predictors of generalisation performance.

Question?
Why, use an objective for the important feature extraction which is a poor predictor of generalisation performance?
When we have provably good predictors (margin, evidence) available in the unimportant classifier?
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A Better approach

1. Combine the feature extraction and classifier learning
2. Choose features which optimise the classifier’s objective

We show how to learn spatio-spectro-temporal feature extractors for classifying ERDs using the max-margin criterion\(^1\)

\(^1\) We have also successfully applied this approach to LR and GP classifiers and MRP/P300 temporal signals.
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Data-visualisation: the ROC-ogram

- Raw data is $dxT$ time-series for $N$ trials
- ROC-ogram: time vs. frequency vs. ROC score for each channel
- allows us to identify where the discriminative information lies
Example raw ROC-ogram: (The good)

Spatial, Spectral, and Temporal discriminative features are subject specific.

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Example raw ROC-ogram: *(The ugly)*

Spatial, Spectral, and Temporal discriminative features are subject specific.
Spatio-Spectro-Temporal feature selection

Would like to **automatically** perform feature selection:

- Spatially,
- Temporally,
- Spectrally

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Learning Feature Extractors

- Start by showing how to learning spatial filters with the max-margin criteria,
- Then extend to learning spatial+spectral+temporal
Spatial Filtering

Volume Conduction – electrodes detect superposition of signals from all over the brain

\[ X = AS \]

Spatial filtering undoes this superposition to re-focus on discriminative signals

\[ y = f_s^T X \]

This is a Blind Source Separation (BSS) problem. Many algorithms are available to solve this problem.

In BCI commonly use a fast, supervised method called Common Spatial Patterns [Koles 1990]
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The Max-margin Objective

- related to an upper bound on generalisation performance
- the basis for the SVM
- finds $w$ such that the minimal distance between classes is maximised
  - in the linear case can be expressed primal objective as,

$$
\min_{w,b} \lambda w^T w + \sum_i \max(0, 1 - y_i (x_i^T w + b))
$$

- for non-linear classification we can simply replace $x_i$ with an explicit feature mapping $\psi(x_i)$
- this is how we include the feature extraction into the classification objective
Max-margin optimised spatial filters

1. Define the feature-space mapping, $\psi$, from time series, $X_i$ to spatially filtered log bandpowers,

$$\psi(X_i, F_s) = \ln(\text{diag}(F_s^\top X_i X_i^\top F_s))$$

where, $F_s = [f_{s1}, f_{s2}, \ldots]$ is the set of spatial filters

2. Include the dependence on $\psi$ explicitly into the classifiers objective, e.g. Linear, Max Margin

$$J_{mm}(X, w, b, F_s) = \lambda w^\top w + \sum_i \max(0, 1 - y_i(\psi(X_i; F_s)^\top w + b))$$

3. Optimise this objective, treating $\psi$'s parameters as additional optimisation variables

Note

Unconstrained optimisation, solve for $w, b, F_s$ directly using CG.
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Unconstrained optimisation, solve for $w, b, F_s$ directly using CG
Adding Spectral/Temporal filters

Very simple to include Spectral/Temporal filtering,.....
.....just modify the feature-mapping $\psi$ to include them.

Let, $f_f$ be a spectral filter, and $f_t$ a temporal filter. Then, the Spatial + Spectral + Temporally filtered band-power is,

$$
\psi(X; f_s, f_f, f_t) = \mathcal{F}^{-1}(\mathcal{F}(f_s^\top XD_t)D_f)(\mathcal{F}^{-1}(\mathcal{F}(f_s^\top XD_t)D_f))^\top
$$

$$
= f_s^\top \mathcal{F}(XD_t)D_f^2 \mathcal{F}(XD_t)^\top f_s^\top / T
$$

where, $\mathcal{F}$ is the Fourier transform, and $D(\cdot) = \text{diag}(f(\cdot))$
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where, \( \mathcal{F} \) is the Fourier transform, and \( D(.) = \text{diag}(f(.)) \)
Filter regularisation

- The filters, $F_s, F_f, F_t$ are unconstrained so may overfit
- We have prior knowledge about the filters shape, e.g.
  - spatial filters tend to be over the motor regions
  - temporal and spectral filters should be smooth
- Include this prior knowledge with quadratic regularisation on the filters,

$$J_{mm} = \lambda w^T w + \sum_i \max(0, 1 - y_i (\ln(\psi(X_i; F_s, F_f, F_t))^T w + b))$$

$$+ \lambda_s \text{Tr}(F_s^T R_s F_s) + \lambda_f \text{Tr}(F_f^T R_f F_f) + \lambda_t \text{Tr}(F_t^T R_t F_t)$$

where, $R(\cdot)$ is a positive definite matrix encoding the prior knowledge
Implementation issues

- Optimising $J_{mm}$ for all the filters directly, results in a “stiff” problem and very slow convergence
- Further, evaluating $\psi(X; f_s, f_f, f_t)$ requires a costly FFT
- Coordinate descent on the filter types solves both these problems,
  1. Spatial optimisation, where, $\psi_s(X, f_s) = f_s^T X_{f,t} X_{f,t}^T f_s$
  2. Spectral optimisation, where $\psi(X; f_f) = \tilde{X}_{s,t} D_f^2 \tilde{X}_{s,t}^T$
  3. Temporal optimisation, where $\psi_t(X, f_t) = X_{s,f} D_t^2 X_{s,f}^T$
  4. Repeat until convergence
- Non-convex problem – seed with good solution found by another method, e.g. CSP or prior knowledge.
Example – Optimisation trajectory for CompIII,Vc

- Noisy CSP seed
- Finds:
  - motor region,
  - 15Hz band,
  - >.5s temporal band
- Finds foot region

Iteration 0: 45% Error
Example – Optimisation trajectory for CompIII, Vc

Iteration 1: 16% Error

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▶ Finds:
  ▶ motor region,
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Example – Optimisation trajectory for CompIII,Vc

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Iteration 2: 3.5% Error

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Iteration 3: 3.5% Error

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Iteration 4: 2.6% Error
Example – Optimisation trajectory for CompIII,Vc

- Noisy CSP seed
- Finds:
  - motor region,
  - 15Hz band,
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- Finds foot region

Iteration 5: 2.6% Error
Example – Optimisation trajectory for CompIII,Vc

- Noisy CSP seed
- Finds:
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Iteration 6: 2.6% Error

Motivation

Automatic Feature Selection

Summary

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Experimental analysis

▶ We show binary classification error from 15 imagined movement subjects:
  ▶ 9 from BCI competitions (Comp 2:IIa, Comp 3:IVa,IVc) and
  ▶ 6 from an internal MPI dataset.
▶ pre-processed by band-pass filtering to .5–45Hz
▶ Baseline performance is from CSP with 2 filters computed on the signal filtered to 7-27Hz.
▶ CSP solution used as the spatial filter seed,
▶ flat seeds used for spectral and temporal filters
Results – Spatial Optimization

- General Improvement in performance; particularly for low numbers of data-points (when overfitting is an issue)
- Huge improvement in a few cases
Results – Spatial + Spectral Optimization

- Further improvement, for the subjects helped before
- Large benefit in a few cases

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Learning optimal EEG features across time, frequency and space.
Results – Spatial + Spectral + Temporal Optimization

100 Training Points
Spatial + Spectral + Temporal

- Further improvements for some subjects
- Slight decrease for others
Summary

- EEG BCI performance depends mainly on learning subject-specific feature extractors
- These can be learnt by direct optimisation of the classification objective (Max-margin)
- Results show significant improvement over independent feature-extractor/classifier learning (better in 12/15 cases)

Future work

- Alternative objective functions – SVM, LR and Gaussian Process objectives implemented already.
- Better priors – particularly for the spatial filters, found by cross-subject learning?
- Other feature/signal types – wavelets, MRPs, P300, etc.
- On-line feature learning
Results – Temporal Signal Extraction

100 Training Points
- learn a rank-1, i.e. 1-spatial + 1-temporal, approximation to the full SVM weight vector
- this regularisation significantly improves classification performance
- and produces readily interpretable results.

200 Training Points

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200 Training Points
Results – Example solutions

- spatially – differential filter between left/right motor regions
- temporally?
- spatially – differential filter between foot and motor regions
- temporally?