Fast Pattern Selection Algorithm for Support Vector Classifiers: Time Complexity Analysis

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Abstract. Training SVM requires large memory and long cpu time when the pattern set is large. To alleviate the computational burden in SVM training, we propose a fast preprocessing algorithm which selects only the patterns near the decision boundary. The time complexity of the proposed algorithm is much smaller than that of the naive M^2 algorithm

1 Introduction

In SVM QP formulation, the dimension of kernel matrix ($M \times M$) is equal to the number of training patterns (M). A standard QP solver has time complexity of order $\mathrm{O}(M^3)$ and a decomposition method has (*the number of iterations*)$\cdot$ $\mathrm{O}(Mq + q^3)$ where q is the size of the working set. Of course, "the number of iterations" is supposed to increase as M increases [3, 4].

One way to circumvent this computational burden is to select only the training patterns, in advance, that are more likely to be support vectors. In a classification problem, the support vectors are distributed near the decision boundary. Therefore, selecting those patterns (potential support vectors) prior to SVM training is quite desirable. Recently, we proposed to select the patterns near the decision boundary based on the neighborhood properties [5]. The first property dictates that a pattern located near the decision boundary tends to have more heterogeneous neighbors. The second property dictates that a pattern on the correct side of the decision boundary tends to belong to the same class as its neighbors. Two measures utilizing these properties reduced the number of patterns significantly, thus reduced the training time. However, a naive algorithm evaluating kNNs for all patterns took $\mathrm{O}(M^2)$, so the pattern selection process itself was time consuming.

In this paper, we propose a fast algorithm. Here, we just compute the kNNs of the patterns near the decision boundary, not all training patterns. The idea comes from another neighborhood property that the neighbors of the pattern located near the decision boundary tend to be located near the decision boundary as well. The time complexity of the fast algorithm is approximately $\mathrm{O}(bM)$, where b is the number of patterns in the "overlap" region around decision boundary. In most practical problems, $b \ll M$ holds.

This paper is structured as follows. In section 2, we propose the fast algorithm which selects the patterns near the decision boundary. In section 3, we provide the time complexity analysis of the algorithm. In section 4, we present the empirical results confirming the time complexity of our algorithm. In the last section, we conclude the paper with the discussion of the limitations.

2 Fast Algorithm based on Neighborhood Properties

The first neighborhood property is that a pattern located near the decision boundary tends to have heterogeneous neighbors. Thus, the degree of pattern $\boldsymbol{x}$'s proximity to the decision boundary can be estimated by "**Neighbors_Entropy($\boldsymbol{x}$)**", which is defined as the entropy of pattern $\boldsymbol{x}$'s k-nearest neighbors' class labels (see Fig. 1). A pattern with a positive Neighbors_Entropy($\boldsymbol{x}$) value is assumed to be located near the decision boundary. *The second neighborhood property* is that a pattern on the correct side of the decision boundary tends to belong to the same class as its neighbors. If a pattern's own label is very different from those of its neighbors, it is likely to be incorrectly labeled. The measure "**Neighbors_Match($\boldsymbol{x}$)**" is defined as the ratio of $\boldsymbol{x}$'s neighbors whose label matches that of $\boldsymbol{x}$. Only those pattern $\boldsymbol{x}$s are selected that satisfy Neighbors_Match($\boldsymbol{x}$) $\geq \beta \cdot \frac{1}{J}$ (J is the number of classes and $0 < \beta \leq 1$) among the patterns with positive Neighbors_Entropy($\boldsymbol{x}$) values.

LabelProbability($\boldsymbol{x}$) {

/* For $\boldsymbol{x}$, calculate the label probabilities of $k\mathbf{NN}(\boldsymbol{x})$ over J classes, $\{C_1, C_2, \ldots, C_J\}$, where $k\mathbf{NN}(\boldsymbol{x})$ is defined as the set of k nearest neighbors of $\boldsymbol{x}$.*/

$k_j = |\{\boldsymbol{x}' \in C_j | \boldsymbol{x}' \in k\mathbf{NN}(\boldsymbol{x})\}|, \;\; j = 1, \ldots, J.$

return $\left(P_j = \frac{k_j}{k}, \forall j\right)$.

}

Neighbors_Entropy($\boldsymbol{x}$) {

/* Calculate the neighbors-entropy of $\boldsymbol{x}$ with its nearest neighbors' labels. In all calculations, $0\, log_J \frac{1}{0}$ is defined to be 0. */

Do **LabelProbability($\boldsymbol{x}$)**.

return $\left(\sum_{j=1}^{J} P_j \cdot log_J \frac{1}{P_j}\right)$.

}

Neighbors_Match($\boldsymbol{x}$) {

/* Calculate the neighbors-match of $\boldsymbol{x}$. j^* is defined as the label of $\boldsymbol{x}$ itself.*/

$j^* = \arg\limits_{j} \{C_j \mid \boldsymbol{x} \in C_j, j = 1, \ldots, J\}$.

Do **LabelProbability($\boldsymbol{x}$)**.

return (P_{j^*}).

}

Fig. 1. Neighbors_Entropy and Neighbors_Match functions

A naive algorithm was presented in [5] where the kNNs of all patterns were evaluated. This algorithm is easy to implement and also runs in a reasonable amount of time as long as the size of training set, M, is relatively small. However, when the size of training set is large, the computational cost increases in proportion to the size. Let us assume that the distance between any two points in d-dimensional space can be computed in $O(d)$. Then finding the nearest neighbors for "each pattern" takes sum of distance computation time DT, $O\left(d(M-1)\right)$, and search time ST, $O\left(k(M-1)\right)$. The total time complexity of the naive algorithm for all patterns, therefore, is $O\left(\ M \cdot (DT+ST)\ \right)$. Roughly speaking, it is $O(M^2)$ if we suppose $d \ll M$ and $k \ll M$. There is a considerable amount of literature on efficient nearest neighbor searching algorithms for large data sets of a high dimension. Most approaches focus on reducing DT or ST. See [1, 2, 6].

Our approach, on the other hand, focuses on reducing the first M of $O(M \cdot M)$. The idea comes from yet *another neighborhood property* that the neighbors of a pattern located near the decision boundary tend to be located near the decision boundary as well. Given a set of randomly selected patterns, we examine the patterns near the decision boundary and their neighbors only. This successive "neighbors" only evaluation of the "current" pattern set is repeated until all the patterns near the decision boundary are chosen and evaluated. A pattern is "expanded" or a pattern's neighbors are evaluated when its Neighbors_Entropy is positive. This "selective kNN expanding" procedure is shown in Fig. 2 using notations displayed in Table 1.

Table 1. Notation

Symbol	Meaning
$\mathbf{D}$	the original training set whose cardinality is M
$\mathbf{D}_{\mathbf{e}}^{i}$	the evaluation set at i-th step
$\mathbf{D}_{\mathbf{o}}^{i}$	a subset of $\mathbf{D}_{\mathbf{e}}^{i}$, the set of patterns to be "expanded" from $\mathbf{D}_{\mathbf{e}}^{i}$ each element of which will compute its k nearest neighbors to constitute the next evaluation set, $\mathbf{D}_{\mathbf{e}}^{i+1}$
$\mathbf{D}_{\mathbf{x}}^{i}$	a subset of $\mathbf{D}_{\mathbf{e}}^{i}$, the set of patterns "not to be expanded" from $\mathbf{D}_{\mathbf{e}}^{i}$, or $\mathbf{D}_{\mathbf{x}}^{i} = \mathbf{D}_{\mathbf{e}}^{i} - \mathbf{D}_{\mathbf{o}}^{i}$
$\mathbf{D}_{\mathbf{s}}^{i}$	the set of "selected" patterns from $\mathbf{D}_{\mathbf{o}}^{i}$ at i-th step
$\mathbf{S}_{\mathbf{o}}^{i}$	the accumulated set of expanded patterns, $\bigcup_{j=0}^{i-1} \mathbf{D}_{\mathbf{o}}^{j}$
$\mathbf{S}_{\mathbf{x}}^{i}$	the accumulated set of non-expanded patterns, $\bigcup_{j=0}^{i-1} \mathbf{D}_{\mathbf{x}}^{j}$
$\mathbf{SS}^{i}$	the accumulated set of selected patterns, $\bigcup_{j=0}^{i-1} \mathbf{D}_{\mathbf{s}}^{j}$ the last of which $\mathbf{SS}^{N}$ is the reduced training pattern set
$k\mathbf{NN}(\boldsymbol{x})$	the set of k nearest neighbors of $\boldsymbol{x}$
$\mathbf{B}$	the set of patterns located in the "overlapped" region characterized by $Neighbors\_Entropy\left(\boldsymbol{x}\right) > 0$
$\mathbf{B}^{+}$	the set of k nearest neighbors of patterns belonging to $\mathbf{B}$

Selective-kNN-Expanding() {

[**0**] Initialize $\mathbf{D}_{\mathbf{e}}^0$ with randomly chosen patterns from $\mathbf{D}$.
Constants k and J are given. Initialize i and various sets as follows:
$i \leftarrow 0, \quad \mathbf{S}_{\mathbf{o}}^0 \leftarrow \emptyset, \quad \mathbf{S}_{\mathbf{x}}^0 \leftarrow \emptyset, \quad \mathbf{SS}^0 \leftarrow \emptyset.$

while $\mathbf{D}_{\mathbf{e}}^i \neq \emptyset$ **do**

[**1**] Choose $\boldsymbol{x}$ satisfying [Expanding Criteria].
$\mathbf{D}_{\mathbf{o}}^i \leftarrow \{\boldsymbol{x} \mid Neighbors\_Entropy\,(\boldsymbol{x}) > 0,\ \boldsymbol{x} \in \mathbf{D}_{\mathbf{e}}^i\}.$
$\mathbf{D}_{\mathbf{x}}^i \leftarrow \mathbf{D}_{\mathbf{e}}^i - \mathbf{D}_{\mathbf{o}}^i.$

[**2**] Select $\boldsymbol{x}$ satisfying [Selecting Criteria].
$\mathbf{D}_{\mathbf{s}}^i \leftarrow \{\boldsymbol{x} \mid Neighbors\_Match\,(\boldsymbol{x}) \geq \beta/J,\ \boldsymbol{x} \in \mathbf{D}_{\mathbf{o}}^i\}.$

[**3**] Update the pattern sets: the expanded, the non-expanded, and the selected.
$\mathbf{S}_{\mathbf{o}}^{i+1} \leftarrow \mathbf{S}_{\mathbf{o}}^i \cup \mathbf{D}_{\mathbf{o}}^i$, $\mathbf{S}_{\mathbf{x}}^{i+1} \leftarrow \mathbf{S}_{\mathbf{x}}^i \cup \mathbf{D}_{\mathbf{x}}^i$, $\mathbf{SS}^{i+1} \leftarrow \mathbf{SS}^i \cup \mathbf{D}_{\mathbf{s}}^i$.

[**4**] Compute the next evaluation set $\mathbf{D}_{\mathbf{e}}^{i+1}$.
$\mathbf{D}_{\mathbf{e}}^{i+1} \leftarrow \bigcup_{\boldsymbol{x} \in \mathbf{D}_{\mathbf{o}}^i} k\mathbf{NN}(\boldsymbol{x}) - (\mathbf{S}_{\mathbf{o}}^{i+1} \cup \mathbf{S}_{\mathbf{x}}^{i+1}).$

[**5**] $i \leftarrow i + 1.$

end

return $\mathbf{SS}^i$**}**

Fig. 2. Selective kNN Expanding algorithm

3 The Time Complexity Analysis of the Fast Algorithm

Now, we show that the fast algorithm terminates within a finite number of steps and its time complexity is significantly smaller than that of the naive algorithm.

Lemma 1. *Different evaluation sets are disjoint:*

$$\mathbf{D}_{\mathbf{e}}^i \cap \mathbf{D}_{\mathbf{e}}^j = \emptyset,\ \forall\, i \neq j. \tag{1}$$

Proof. Consider step [4] of the algorithm shown in Fig.2,

$$\mathbf{D}_{\mathbf{e}}^i = \left(\bigcup_{\boldsymbol{x} \in \mathbf{D}_{\mathbf{o}}^{i-1}} k\mathbf{NN}(\boldsymbol{x}) \right) - (\mathbf{S}_{\mathbf{o}}^i \cup \mathbf{S}_{\mathbf{x}}^i). \tag{2}$$

Since $\mathbf{S}_{\mathbf{o}}^i$ and $\mathbf{S}_{\mathbf{x}}^i$ are defined as $\left(\bigcup_{j=0}^{i-1} \mathbf{D}_{\mathbf{o}}^j \right)$ and $\left(\bigcup_{j=0}^{i-1} \mathbf{D}_{\mathbf{x}}^j \right)$ respectively,

$$\mathbf{S}_{\mathbf{o}}^i \cup \mathbf{S}_{\mathbf{x}}^i = \bigcup_{j=0}^{i-1} \left(\mathbf{D}_{\mathbf{o}}^i \cup \mathbf{D}_{\mathbf{x}}^i \right) = \left(\bigcup_{j=0}^{i-1} \mathbf{D}_{\mathbf{e}}^j \right). \tag{3}$$

By replacing $\left(\mathbf{S}_{\mathbf{o}}^i \cup \mathbf{S}_{\mathbf{x}}^i \right)$ in Eq.(2) with Eq.(3), we get

$$\mathbf{D}_{\mathbf{e}}^i = \left(\bigcup_{\boldsymbol{x} \in \mathbf{D}_{\mathbf{o}}^{i-1}} k\mathbf{NN}(\boldsymbol{x}) \right) - \left(\bigcup_{j=0}^{i-1} \mathbf{D}_{\mathbf{e}}^j \right). \tag{4}$$

Eq.(4) clearly shows that $\mathbf{D}_{\mathbf{e}}^i$ does not share patterns with any of its earlier sets.∎

Lemma 2. *The union of all* $\mathbf{D}_{\mathbf{e}}^{i}$*'s is equivalent to the set of kNN's of the union of all* $\mathbf{D}_{\mathbf{o}}^{i}$*'s.*

$$\left(\bigcup_{i=1}^{n}\mathbf{D}_{\mathbf{e}}^{i}\right)=\left(\bigcup_{\boldsymbol{x}\in\mathbf{D}_{\mathbf{o}}^{0}\cup\mathbf{D}_{\mathbf{o}}^{1}\cup\cdots\cup\mathbf{D}_{\mathbf{o}}^{n-1}}k\mathbf{NN}(\boldsymbol{x})\right). \tag{5}$$

Proof. From Eq.(4) in Lemma 1, we get

$$\bigcup_{i=1}^{n}\mathbf{D}_{\mathbf{e}}^{i}=\bigcup_{i=1}^{n}\left(\bigcup_{\boldsymbol{x}\in\mathbf{D}_{\mathbf{o}}^{i-1}}k\mathbf{NN}(\boldsymbol{x})\right)-\bigcup_{i=1}^{n}\left(\bigcup_{j=0}^{i-1}\mathbf{D}_{\mathbf{e}}^{j}\right). \tag{6}$$

Since in general

$$\left(\bigcup_{\boldsymbol{x}\in\mathbf{A}_1}k\mathbf{NN}(\boldsymbol{x})\right)\bigcup\left(\bigcup_{\boldsymbol{x}\in\mathbf{A}_2}k\mathbf{NN}(\boldsymbol{x})\right)=\left(\bigcup_{\boldsymbol{x}\in\mathbf{A}_1\cup\mathbf{A}_2}k\mathbf{NN}(\boldsymbol{x})\right) \tag{7}$$

holds, we get

$$\left(\bigcup_{i=1}^{n}\mathbf{D}_{\mathbf{e}}^{i}\right)=\left(\bigcup_{\boldsymbol{x}\in\mathbf{D}_{\mathbf{o}}^{0}\cup\mathbf{D}_{\mathbf{o}}^{1}\cup\cdots\cup\mathbf{D}_{\mathbf{o}}^{n-1}}k\mathbf{NN}(\boldsymbol{x})\right)-\left(\bigcup_{i=0}^{n-1}\mathbf{D}_{\mathbf{e}}^{i}\right). \tag{8}$$

If we union $\left(\bigcup_{i=0}^{n-1}\mathbf{D}_{\mathbf{e}}^{i}\right)$ to both sides of Eq.(8), then

$$\left(\bigcup_{i=0}^{n}\mathbf{D}_{\mathbf{e}}^{i}\right)=\left(\bigcup_{\boldsymbol{x}\in\mathbf{D}_{\mathbf{o}}^{0}\cup\mathbf{D}_{\mathbf{o}}^{1}\cup\cdots\cup\mathbf{D}_{\mathbf{o}}^{n-1}}k\mathbf{NN}(\boldsymbol{x})\right)\bigcup\left(\bigcup_{i=0}^{n-1}\mathbf{D}_{\mathbf{e}}^{i}\right) \tag{9}$$

results. Since $\mathbf{D}_{\mathbf{e}}^{i}\subseteq\bigcup_{\boldsymbol{x}\in\mathbf{D}_{\mathbf{o}}^{i-1}}k\mathbf{NN}(\boldsymbol{x}),\ i=1,\ldots,n,\ \left(\bigcup_{i=1}^{n-1}\mathbf{D}_{\mathbf{e}}^{i}\right)$, the last $n-1$ components of the second factor of the right hand side may vanish. Then, we finally have

$$\left(\bigcup_{i=1}^{n}\mathbf{D}_{\mathbf{e}}^{i}\right)\bigcup\mathbf{D}_{\mathbf{e}}^{0}=\left(\bigcup_{\boldsymbol{x}\in\mathbf{D}_{\mathbf{o}}^{0}\cup\mathbf{D}_{\mathbf{o}}^{1}\cup\cdots\cup\mathbf{D}_{\mathbf{o}}^{n-1}}k\mathbf{NN}(\boldsymbol{x})\right)\bigcup\mathbf{D}_{\mathbf{e}}^{0}. \tag{10}$$

If we consider only the relationship after the first iteration, then $\mathbf{D}_{\mathbf{e}}^{0}$ from both sides of Eq.(10) is not to be included. Now, the lemma is proved. ∎

Lemma 3. *Every expanded set* $\mathbf{D}_{\mathbf{o}}^{i}$ *is a subset of* $\mathbf{B}$*, the set of patterns in the overlapped region.*

$$\mathbf{D}_{\mathbf{o}}^{i}\subseteq\mathbf{B},\ \forall i \tag{11}$$

Proof. Recall that in the proposed algorithm, $\mathbf{D}_{\mathbf{o}}^{i}$ is defined as

$$\mathbf{D}_{\mathbf{o}}^{i}=\{\boldsymbol{x}\mid Neighbors\_Entropy\,(\boldsymbol{x})>0,\ \boldsymbol{x}\in\mathbf{D}_{\mathbf{e}}^{i}\}. \tag{12}$$

Compare it with the definition of $\mathbf{B}$

$$\mathbf{B}=\{\boldsymbol{x}\mid Neighbors\_Entropy\,(\boldsymbol{x})>0,\ \boldsymbol{x}\in\mathbf{D}\}. \tag{13}$$

Since $\mathbf{D}_{\mathbf{e}}^{i}$'s are subsets of $\mathbf{D}$, $\mathbf{D}_{\mathbf{o}}^{i}$'s are subsets of $\mathbf{B}$. ∎

Lemma 4. *Different expanded sets $\mathbf{D}_\mathbf{o}^i$'s are disjoint.*

$$\mathbf{D}_\mathbf{o}^i \cap \mathbf{D}_\mathbf{o}^j = \emptyset,\ \forall i \neq j \tag{14}$$

Proof. Every expanded set is a subset of the evaluation set by definition (see step[1] in the algorithm)

$$\mathbf{D}_\mathbf{o}^i \subseteq \mathbf{D}_\mathbf{e}^i,\ \forall i. \tag{15}$$

By Lemma 1, $\mathbf{D}_\mathbf{e}^i$'s are disjoint from others for all i's. Therefore, their respective subsets are disjoint, too. ∎

Theorem 1. *(Termination of the Algorithm) If the while loop of the proposed algorithm exits after N iterations, then N is finite.*

Proof. We show that $N < \infty$. Inside the while-loop of the algorithm(Fig.2), condition $\mathbf{D}_\mathbf{e}^{i+1} \neq \emptyset$ holds. Therefore, $\mathbf{D}_\mathbf{o}^i \neq \emptyset,\ i = 0, \ldots, N-1$. That means $n(\mathbf{D}_\mathbf{o}^i) \geq 1,\ i = 0, \ldots, N-1$. Since $\mathbf{S}_\mathbf{o}^i$ is defined as $\bigcup_{j=0}^{i-1} \mathbf{D}_\mathbf{o}^j$, and $\mathbf{D}_\mathbf{o}^j$'s are disjoint (Lemma 4), we get

$$n(\mathbf{S}_\mathbf{o}^i) = \sum_{j=0}^{i-1} n(\mathbf{D}_\mathbf{o}^j). \tag{16}$$

Since $n(\mathbf{D}_\mathbf{o}^i) \geq 1,\ i = 0, \ldots, N-1$, $n(\mathbf{S}_\mathbf{o}^i)$ is monotonically increasing. In the meantime, the union of all the $\mathbf{D}_\mathbf{o}^j$'s generated in the while loop is bounded by $\mathbf{B}$ (Lemma 3). So, we obtain

$$\bigcup_{j=0}^{N-1} \mathbf{D}_\mathbf{o}^j \subseteq \mathbf{B}. \tag{17}$$

Now, Lemma 4 leads us to

$$\sum_{j=0}^{N-1} n(\mathbf{D}_\mathbf{o}^j) \leq n(\mathbf{B}). \tag{18}$$

Combination of Eq.(16) and Eq.(18) results in

$$n(\mathbf{S}_\mathbf{o}^N) \leq n(\mathbf{B}). \tag{19}$$

Since $n(\mathbf{B}) \ll M$ and finite, $n(\mathbf{S}_\mathbf{o}^N)$ is finite. Thus, N is finite. ∎

Theorem 2. *(The Number of Pattern Evaluation) The number of patterns whose kNNs are evaluated is $\left(r \cdot n(\mathbf{B}^C) + n(\mathbf{B}^+)\right)$, where $\mathbf{B}^C$ is the complement set of $\mathbf{B}$ or $\mathbf{D} - \mathbf{B}$, and r is the proportion of initial random sampling, $(0 < r < 1)$.*

Proof. The number of patterns whose kNNs are evaluated is denoted as $\sum_{i=0}^{N} n(\mathbf{D}_\mathbf{e}^i)$. Let us first consider cases from $i = 1$ to N. We have

$$\sum_{i=1}^{N} n(\mathbf{D}_\mathbf{e}^i) = n\left(\bigcup_{i=1}^{N} \mathbf{D}_\mathbf{e}^i\right) \qquad \text{by Lemma 1}$$

$$= n\left(\bigcup_{\boldsymbol{x}\in\mathbf{D}_\mathbf{o}^0\cup\mathbf{D}_\mathbf{o}^1\cup\cdots\cup\mathbf{D}_\mathbf{o}^{N-1}} k\mathbf{NN}(\boldsymbol{x})\right) \quad \text{by Lemma 2}$$

$$\leq n\left(\bigcup_{\boldsymbol{x}\in\mathbf{B}} k\mathbf{NN}(\boldsymbol{x})\right) \quad \text{by Lemma 3}$$

$$= n(\mathbf{B}^+). \tag{20}$$

Let us include the case of $i = 0$.

$$\sum_{i=0}^{N} n(\mathbf{D}_\mathbf{e}^i) \leq n(\mathbf{D}_\mathbf{e}^0) + n(\mathbf{B}^+), \tag{21}$$

where $n(\mathbf{D}_\mathbf{e}^0)$ is approximately $r \cdot n(\mathbf{D})$ because $\mathbf{D}_\mathbf{e}^0$ is randomly chosen from $\mathbf{D}$. In the meantime, some patterns of $\mathbf{D}_\mathbf{e}^0$ are already counted in $n(\mathbf{B}^+)$. The number of those pattern amounts to $n(\mathbf{D}_\mathbf{o}^0)$ since $\mathbf{D}_\mathbf{o}^0 = \{\boldsymbol{x} \mid Neighbors\_Entropy\,(\boldsymbol{x}) > 0,\ \boldsymbol{x} \in \mathbf{D}_\mathbf{e}^0\}$ and $\mathbf{D}_\mathbf{o}^0 \subseteq \mathbf{B} \subseteq \mathbf{B}^+$. To get a tighter bound, therefore, we calculate $n(\mathbf{D}_\mathbf{e}^0 - \mathbf{D}_\mathbf{o}^0)$ ($\leq n(\mathbf{D}_\mathbf{e}^0)$)

$$\begin{aligned} n(\mathbf{D}_\mathbf{e}^0 - \mathbf{D}_\mathbf{o}^0) &= n(\mathbf{D}_\mathbf{e}^0) - n(\mathbf{D}_\mathbf{o}^0) \\ &\approx n(\mathbf{D}_\mathbf{e}^0) - n(\mathbf{D}_\mathbf{e}^0) \cdot \frac{n(\mathbf{B})}{n(\mathbf{D})} \\ &= r \cdot n(\mathbf{D}) - r \cdot n(\mathbf{D}) \cdot \frac{n(\mathbf{B})}{n(\mathbf{D})} \\ &= r \cdot n(\mathbf{B}^C), \end{aligned} \tag{22}$$

where $\mathbf{D}_\mathbf{o}^0 \subseteq \mathbf{D}_\mathbf{e}^0$ and $n(\mathbf{D}_\mathbf{e}^0 - \mathbf{D}_\mathbf{o}^0)$ denotes the number of the patterns which do not belong to $\mathbf{B}$. Thus, we get the following bound by Eq.(20) and Eq.(22):

$$\sum_{i=0}^{N} n(\mathbf{D}_\mathbf{e}^i) \leq r \cdot n(\mathbf{B}^C) + n(\mathbf{B}^+). \qquad \blacksquare$$

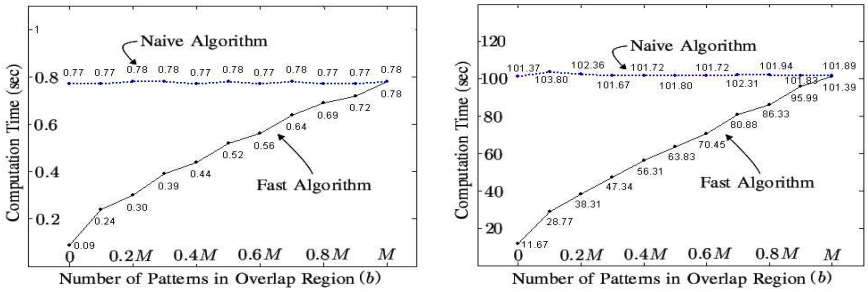
The time complexity of the fast algorithm is $(r \cdot b^C + b^+) \cdot M$ where $b^C = n(\mathbf{B}^C)$ and $b^+ = n(\mathbf{B}^+)$. Practically, b^C is almost as large as M, i.e., $b^C \approx M$. But the initial sampling ratio r is usually quite small, i.e., $r \ll 1$. Thus the first term $r \cdot b^C M$ may be assumed to be insignificant. In most real world problems, b^+ is just slightly larger than b, thus the second term $b^+ M$ can be approximated to bM. In short, $(r \cdot b^c + b^+)M$ can be simplified as bM, which of course is much smaller than M^2 since $b \ll M$.

4 Experimental Results

The fast algorithm runs in bM, roughly speaking. We now show whether the complexity stands in the practical situations through experiments. A total of M patterns, half of M from each class, were randomly generated from a pair of two-dimensional uniform distributions:

$$C_j = \left\{\boldsymbol{x} \mid U\left(\left[\begin{matrix}-1\\ (\frac{-1+(-1)^{j+1}}{2} + \frac{(-1)^j b}{2M})\end{matrix}\right] < \boldsymbol{x} < \left[\begin{matrix}1\\ (\frac{1+(-1)^{j+1}}{2} + \frac{(-1)^j b}{2M})\end{matrix}\right]\right)\right\},\ j = 1, 2.$$

We set b to every decile of M, i.e. b= 0, $0.1M$, $0.2M$, $\cdots$, $0.9M$, M. Fig. 3 shows the actual computation time for various values of b when (a) M=1,000 and (b) M=10,000, respectively. Compare with the Naive algorithm's computation time that is constant regardless of b. They clearly show that computation time is exactly proportional to b.



(a) $M = 1,000$ **(b)** $M = 10,000$

Fig. 3. Actual computation time for various values of b

5 Conclusion

We proposed a fast pattern selection algorithm which takes $O(bM)$ time. Currently, however, there are two limitations. First, the proposed algorithm was developed under the assumption that the classes are overlapped. Therefore, if one class is remote and clearly separable from the other, an empty set will be returned as a selected pattern set. Second, the number of neighbors, k, was empirically set to 4 in the experiment. A more scientific method is currently under investigation.

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