

FALK LIEDER

Précis of BEYOND BOUNDED RATIONALITY: REVERSE-ENGINEERING AND ENHANCING HUMAN INTELLIGENCE

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Bad decisions can have devastating consequences: There is a vast body of literature claiming that human judgment and decision-making are riddled with numerous systematic violations of the rules of logic, probability theory, and expected utility theory. The discovery of these *cognitive biases* in the 1970s (Tversky & Kahneman, 1974) made people question the concept of Homo sapiens as the rational animal, profoundly shaking the foundations of economics and rational models in the cognitive, neural, and social sciences. Four decades later, these disciplines still lack a rigorous theoretical foundation for explaining and remedying people's cognitive biases. To solve this problem, my dissertation offers a mathematically precise theory of bounded rationality and demonstrates how it can be leveraged to elucidate the cognitive mechanisms of judgment and decision-making (Part 1) and to help people make better decisions (Part 2).

Part 1: Bounded Rationality Revisited

The first part of my dissertation revisits the debate about human rationality by redefining what it means to be rational. In Chapter 1, I introduce the first realistic and mathematically precise normative standard of human rationality: *resource rationality*. In Chapters 2 and 3, I apply this new theory of rationality to show that the human mind is not nearly as irrational as the literature on cognitive biases in judgment and decision-making might suggest. Complementing the rational reinterpretations of multiple seemingly irrational heuristics proposed in Chapters 2 and 3, Chapter 4 presents evidence that people decide when to use which heuristic according to a rational cost-benefit analysis. Chapter 5 shows that this capacity for rational strategy selection is acquired through learning; it presents evidence that, when put in a new environment, people gradually learn to make increasingly more rational use of their finite time and bounded cognitive resources. Chapter 6 extends the model of strategy selection learning into a computational model of how people might discover their cognitive strategies and a machine learning method for discovering them. This method can be used to partially automate the process of cognitive

modeling. Comparing its predictions to human data paints a brighter and more nuanced picture of human rationality than previous work.

Chapter 1: Resource Rationality

To develop a theoretical framework that can account for cognitive biases, I start from the assumption that human cognition is fundamentally constrained by limited time and the brain's finite computational resources. Based on this assumption, I redefine human rationality as reasoning and deciding according to cognitive strategies that make the best possible use of the mind's limited resources. I draw on the bounded optimality framework developed in the artificial intelligence literature (Russel & Subramanian, 1995) to translate this definition into a mathematically precise theory of bounded rationality called *resource rationality* (Lieder & Griffiths, 2019; see **Error! Reference source not found.**).

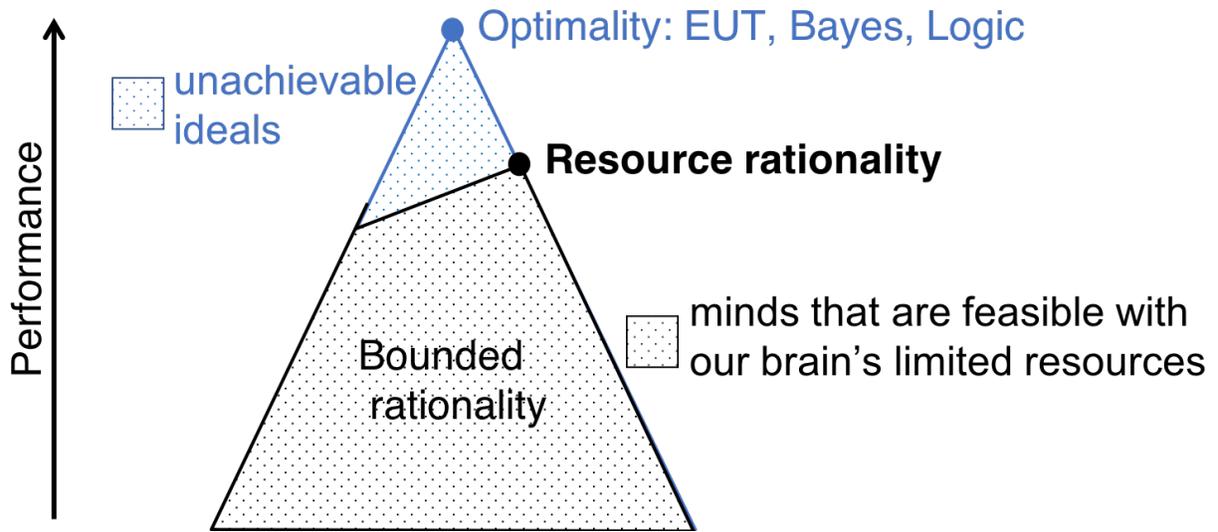


Figure 1: Resource rationality and its relationship to optimality and Tversky and Kahneman's concept of bounded rationality (Tversky & Kahneman, 1974). The horizontal dimension corresponds to alternative cognitive mechanisms that achieve the same level of performance. Each point inside the pyramid is a possible mind. The black ones are feasible with the brain's bounded cognitive resources whereas the blue ones would require more computational resources than the human brain can offer. The thick black line symbolizes the bounds entailed by people's limited cognitive resources.

In brief, for an agent whose brain B is capable of executing the heuristics H_B the resource rational heuristic h^* for the environment E is

$$h^*(b_0, B, E) = \operatorname{argmax}_{h \in H_B} \mathbb{E}_{P(\text{result}|s_0, h, E, B)}[u(\text{result})] - \mathbb{E}_{t_h, \rho, \lambda | h, s_0, B, E}[\text{cost}(t_h, \rho, \lambda)], \quad (3)$$

where $u(\text{result})$ is the agent's subjective utility u of the outcomes (result) of the choices made by the heuristic h , $s_0 = (o, b_0)$ comprises the observed information about the initial state of the external world (o) and the agent's initial internal state b_0 , and $\text{cost}(t_h, \rho)$ denotes the total opportunity cost of investing the cognitive resources ρ used or blocked by the heuristic h for the duration t_h of its execution. Both the result of applying the heuristic and its execution time depend on the situation in which it is applied. The expected values (\mathbb{E}) weigh the utility and cost

for each possible situation by their posterior probability given the environment E and the observed characteristics of the current situation (o). This definition lets us derive cognitive mechanisms from a specification of their function and a model of the cognitive resources that are available to realize this function. This makes it possible to advance rational analysis (Anderson, 1999) to the algorithmic level of analysis (Marr, 1982) by following the methodology summarized in Box 1 (Lieder & Griffiths, 2019). We can thereby leverage powerful and elegant optimality principles that have allowed computational cognitive scientists to characterize cognitive functions and predict human behavior to discover cognitive mechanisms. Throughout my Ph.D., I have applied resource rational analysis to elucidate the mechanisms of judgment and decision-making, revisit the debate about human rationality, and enhance and augment human rationality.

1. Start with a computational-level (i.e., functional) description of an aspect of cognition formulated as a problem and its solution.
2. Posit which class of algorithms the mind's computational architecture might use to approximately solve this problem, a cost in computational resources used by these algorithms, and the utility of more accurately approximating the correct solution.
3. Find the algorithm in this class that optimally trades off resources and approximation accuracy (Equation 1).
4. Evaluate the predictions of the resulting rational process model against empirical data.
5. Refine the computational-level theory (Step 1) or assumed computational architecture and its constraints (Step 2) to address significant discrepancies, derive a refined resource rational model, and then reiterate or stop if the model's assumptions are already sufficiently realistic.

Box 1. The five steps of resource rational analysis. Note that a resource rational analysis may stop in Step 5 even when human performance substantially deviates from the resource rational predictions as long as reasonable attempts have been made to model the constraints accurately based on the available empirical evidence. Furthermore, refining the assumed computational architecture can also include modeling how the brain might approximate the postulated algorithm.

Chapters 2 and 3: A Resource Rational Perspective on Anchoring and Availability Biases

In their groundbreaking paper *Judgment under uncertainty: heuristics and biases*, Tversky and Kahneman (1974) rebutted human rationality by demonstrating three ways that people's judgments and decisions systematically violate the rules of logic and probability theory: the anchoring bias, the availability bias, and representativeness. Complementing the rational reinterpretation of representativeness (Tenenbaum & Griffiths, 2001), I refute the predominant view that anchoring and availability biases demonstrate that people are fundamentally irrational by showing that anchoring and availability reflect the rational use of finite time and bounded cognitive resources.

Anchoring is the phenomenon that people's estimates of numerical quantities are biased toward potentially irrelevant numbers that were on their minds just before they started to estimate. Contrary to the common view that anchoring is irrational, I successfully modelled adjustment as a process that is rational in the sense that i) it converges to the Bayes optimal inference if sufficiently many adjustments are made (see Figure 2a), and ii) the number of

adjustments is determined by a rational cost-benefit analysis that minimizes the sum of error cost and time cost (see Figure 2b). Critically, according to this rational cost-benefit analysis, the optimal number of adjustments is often so small that the adjusted estimate remains biased toward the initial value; i.e., high anchors lead to larger estimates than low anchors (see **Error! Reference source not found.**a). A series of simulations showed that this model can explain the effects of both provided anchors and self-generated anchors as well as the effects of cognitive load, time pressure, alcohol, anchor extremity, uncertainty, and knowledge; it also explains the effect of incentives and their interaction with three different moderators (Lieder, Griffiths, Huys, & Goodman, 2017a). Furthermore, our model predicted that the anchoring bias should increase with time cost and decrease with error cost whether the anchors were provided or self-generated. We confirmed these predictions in two behavioral experiments (Lieder, Griffiths, Huys, & Goodman, 2017b). This suggests that anchoring is a consequence of the rational use of finite time.

Availability bias is the phenomenon that people’s judgments and decisions are overly swayed by events that come to mind easily. As a consequence, people overestimate the frequency of extremely bad events (e.g., terrorism) and extremely good events (e.g., winning the lottery) and overweight them in their decisions. This is irrational from the perspective of expected utility theory and probability theory. But my resource rational analysis of decision-making under uncertainty suggested that these phenomena can be understood as the consequence of people allocating their limited attention to the most important eventualities. Concretely, I showed that the optimal way to estimate an action’s expected utility with a limited number of simulations is to overrepresent events in proportion to how extremely good or how extremely bad they are: If the possible outcome o with utility $u(o)$ occurs with probability $p(o)$, the optimal decision mechanism’s propensity to simulate this event is as if its probability were

$$q_{UWS}(o) \propto p(o) \cdot |u(o) - \bar{u}|, \quad (2)$$

where \bar{u} is the average utility of previously experienced events. The optimal decision mechanism – utility-weighted sampling (UWS) – subsequently corrects for this bias as well as possible, but if the number of simulations is finite, some bias will inevitably remain. This predicts that the degree to which people overestimate the frequency of an event should be proportional to the extremity of its utility ($|u(o) - \bar{u}|$). The experimental findings shown in Figure 3 confirmed this prediction. In addition to explaining people’s cognitive bias to overestimate the frequency of extreme events, the utility-weighted sampling model provided a unifying explanation for 12 additional phenomena, including cognitive biases in memory recall, learning, decisions from experience, and decisions from description (Lieder, Griffiths, & Hsu, 2018).

By showing that availability biases and the anchoring bias are consistent with the rational use of limited resources, my analysis provides a rational reinterpretation of cognitive biases that were once interpreted as hallmarks of human irrationality. This suggests that it is time to revisit the debate about human rationality with a more realistic normative standard of resource rationality.

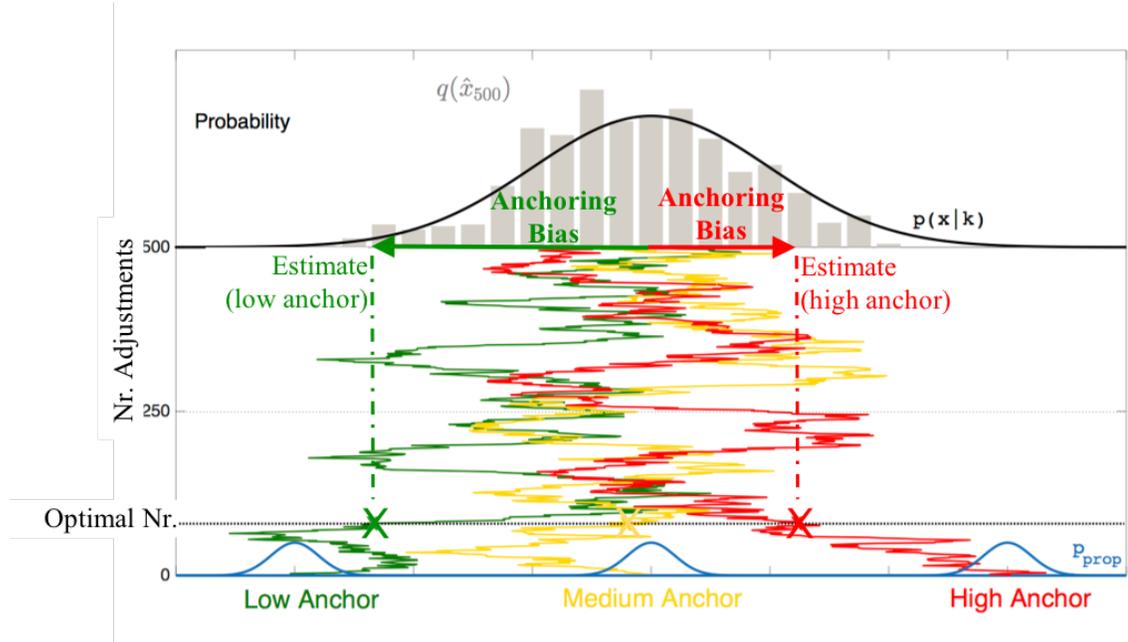
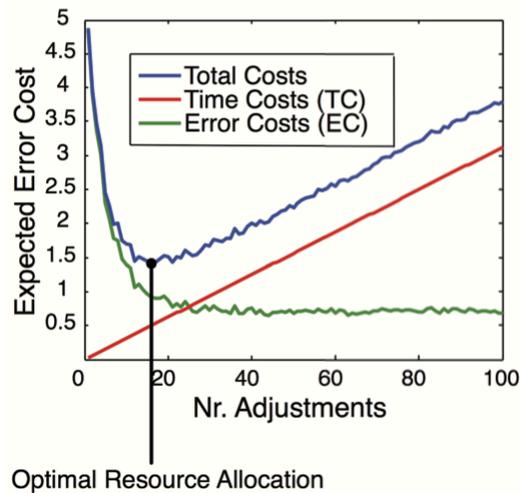
a**b**

Figure 2: Our resource rational model of the anchoring bias.

a: Illustration of the adjustment process. The three jagged lines are examples of sequence of estimates that the adjustment process generates starting from a low, medium, and high anchor, respectively. The top of the figure shows that after 250 adjustments the relative frequencies of the estimates agree with the target distribution $p(x|k)$ regardless of the anchor. This is because the influence of the anchor vanishes as the number of adjustments increases. Yet, when the number of adjustments is small, the estimates are still biased toward their initial values. The optimal number of adjustments is very low, as illustrated by the dotted line. Consequently, the resulting estimates indicated by the green, yellow, and red crosses are still biased toward their respective anchors.

b: Illustration of the rational cost-benefit analysis that determines the optimal number of adjustments.

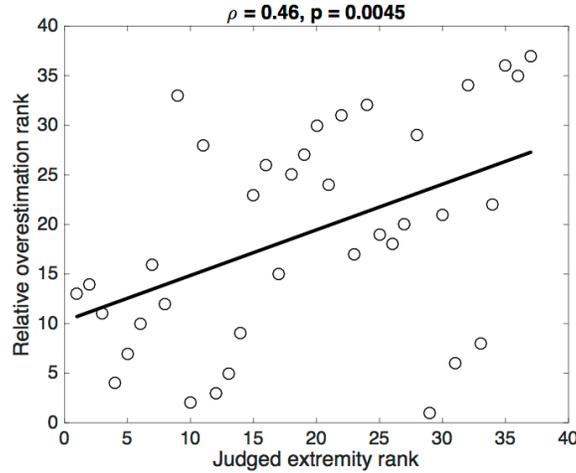


Figure 3: Relative overestimation increases with perceived extremity. Each circle represents one event's average ratings.

Chapter 4: A Rational Solution to the Strategy Selection Problem

Since people are equipped with multiple heuristics, a complete normative theory of bounded rationality also must answer the question of when each should be used. I addressed this question by developing a rational theory of strategy selection (Lieder & Griffiths, 2017). In brief, optimal strategy selection should choose the heuristic h^* that achieves the best tradeoff between the expected reward (R) of the resulting decisions and the expected opportunity cost of the time T it takes to execute the strategy, that is,

$$h^* = \operatorname{argmax}_h \mathbb{E}[R|\text{problem}, h] - \gamma \cdot \mathbb{E}[T|\text{problem}, h], \quad (3)$$

where γ is the agent's opportunity cost per unit time. As illustrated in A, I propose that the mind approximates the optimal solution to this problem by learning to efficiently predict the costs and benefits of each strategy by a weighted sum of features f_1, \dots, f_n of the problem to which it would be applied. This is accomplished by learning the weights $w_{1,h}, \dots, w_{n,h}$ that strategy h assigns to the features from experience through Bayesian regression. Given a set of learned weights, the cost-benefit analysis becomes a single forward pass in the neural network illustrated in a.

As illustrated in Figure 4b, our rational model of strategy selection learning was the only model that could account for people's adaptive strategy choices in a sorting task (Lieder & Griffiths, 2017). Moreover, this model captures the variability, contingency, and change of people's strategy choices in the domains of decision-making, mental arithmetic, and problem-solving; it even captures aspects of children's cognitive development in the domain of mental arithmetic. Most importantly, our model solves the strategy selection problem that was perhaps the last critical missing piece in the adaptive toolbox theory of bounded rationality (Gigerenzer & Selten, 2002); this is an important step toward understanding human rationality. In addition, our model of how people learn when to use which cognitive strategy has advanced artificial intelligence by outperforming state-of-the-art methods for algorithm selection (Lieder, et al., 2014).

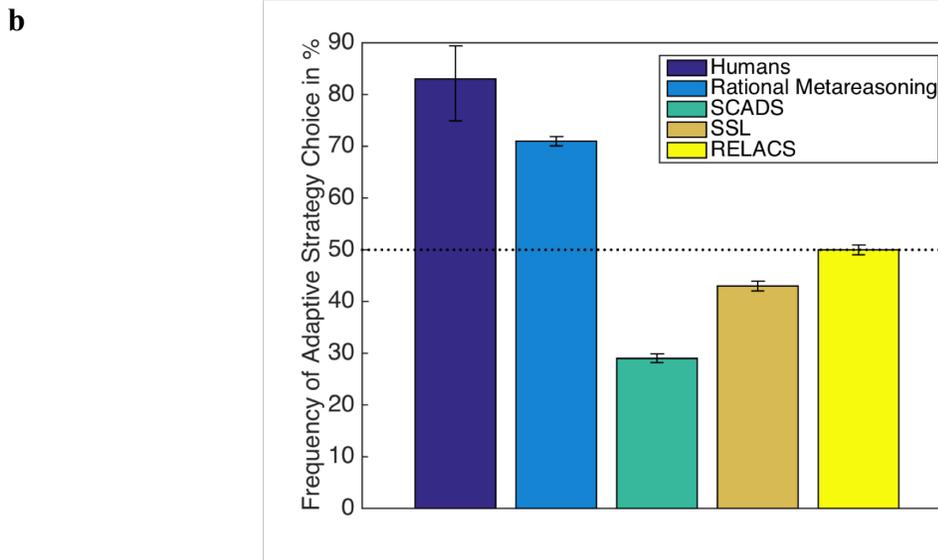
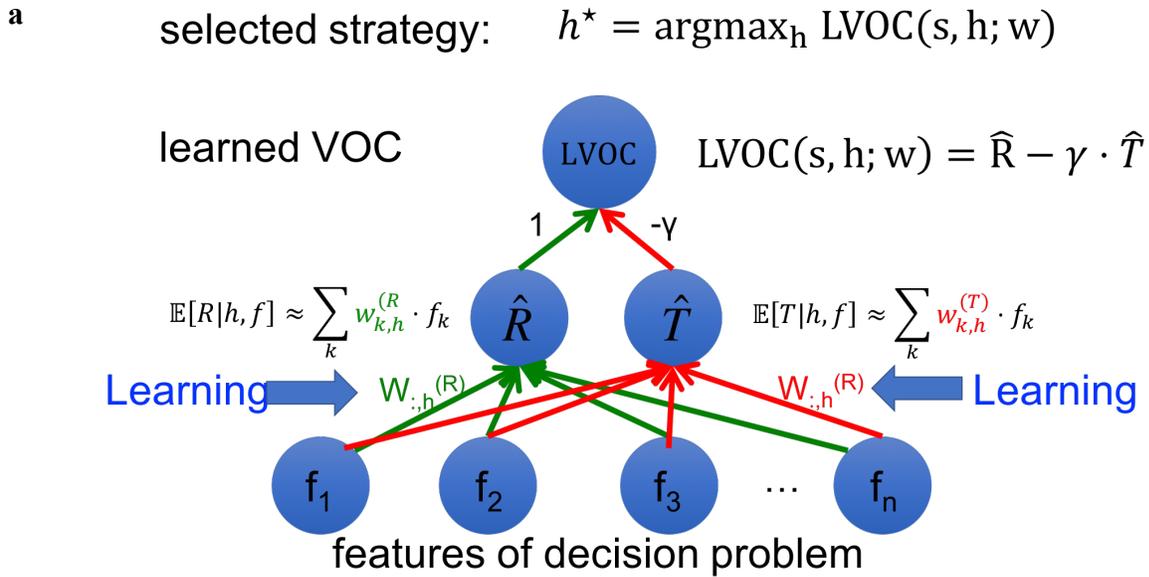


Figure 4: Our rational metareasoning model of strategy selection captures people's capacity for adaptive strategy selection better than any previous models.

Chapter 5: People Gradually Learn to Make Increasingly More Rational Use of their Cognitive Resources

According to our rational model of strategy selection learning, people should gradually learn to select the heuristic with the best possible speed-accuracy trade-off by building a predictive model of its performance. I confirmed this prediction in a series of three experiments showing that when people initially think too little they learn to think more, that when people initially think too much they learn to think less, and that people learn to select their strategies in an adaptive manner, respectively (Lieder & Griffiths, 2017). Figure 5 illustrates our model's ability to capture people's capacity to gradually adapt their strategy choices (e.g., how often they use the Take-The-Best heuristic) to the structure of the environment.

These results and subsequent findings (Krueger, Lieder, & Griffiths, 2017) suggest that people gradually learn to make increasingly more rational use of their finite time and bounded cognitive resources. This means that human rationality is not fixed but malleable, and even when people initially use irrational heuristics, their levels of resource rationality tend to increase over time as they learn to adapt to the structure of their environments.

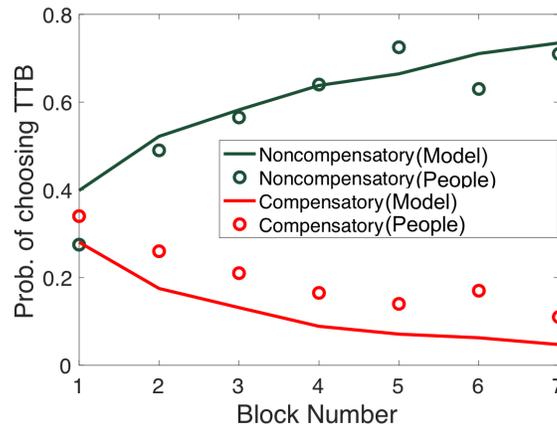


Figure 5: The fit of our rational model of strategy selection learning to the data from Rieskamp and Otto (2006). The Y-axis shows the frequency with which people and the model selected the Take-The-Best heuristic (TTB) depending on whether the environment favored strategies that, like TTB, focus on a small number of attributes (noncompensatory) versus strategies that integrate all of the information (compensatory).

In subsequent work, we investigated where people’s decision strategies come from (Krueger*, Lieder*, & Griffiths, 2017). We found that the mechanisms by which people discover and continuously refine their decision strategies can be understood as metacognitive reinforcement learning. Our metacognitive reinforcement learning model captured the temporal dynamics of how participants learned how to plan what to do. Moreover, our model also captured how the structure of the learning environment and the addition of feedback affected these learning dynamics. Building on this success, we extended our model of metacognitive reinforcement learning into a machine learning method for automatic strategy discovery.

Chapter 6: An Automatic Method for Strategy Discovery

Having defined a resource rational heuristic as the solution to a constrained optimization problem (see Equation 1) makes it possible to leverage methods from artificial intelligence to discover optimal cognitive strategies automatically. In Chapter 6 of my dissertation, I introduced a general computational framework (i.e., meta-level Markov decision processes; see Figure 6A) where the formerly manual mathematical derivation of resource rational heuristics can be automated. Extending the metacognitive reinforcement learning model of how people learn how to decide that is presented in Chapter 5 led to two strategy discovery algorithms: the Bayesian SARSA algorithm (Lieder, Krueger, & Griffiths, 2017) and the Bayesian meta-level policy search algorithm (Lieder, Callaway, Gul, Krueger, & Griffiths, 2017) which advanced artificial intelligence by outperforming previous metareasoning methods (Callaway, Gul, Krueger, Griffiths, & Lieder, 2018).

As a first illustration, we applied automatic strategy discovery to multi-alternative risky choice (see Figure 6b; Lieder, Krueger, & Griffiths, 2017). In addition to rediscovering well-known fast-and-frugal heuristics as resource rational strategies for specific environments, the

Bayesian SARSA algorithm also discovered a previously unknown rational heuristic: the SAT-TTB heuristic illustrated in Figure 6b. This heuristic is a satisficing version of the Take-The-Best heuristic and is optimal in environments where some outcomes are much more likely than others and the stakes are relatively low (see Figure 6b). A subsequent experiment confirmed that people use this heuristic very frequently in environments where it is resource rational to do so (Lieder, Krueger, & Griffiths, 2017). Moreover, automatic strategy discovery correctly predicted which heuristic people used most often in each of four different environments.

Automatic strategy discovery makes it possible to systematically assess the extent to which human cognition is resource rational. Applying our method to different multi-alternative risky choice environments and evaluating human decision-making against resource rational heuristics, we found that, on average, human decision-making is at most 88% as resource rational as it could be (Lieder, Krueger, & Griffiths, 2017).

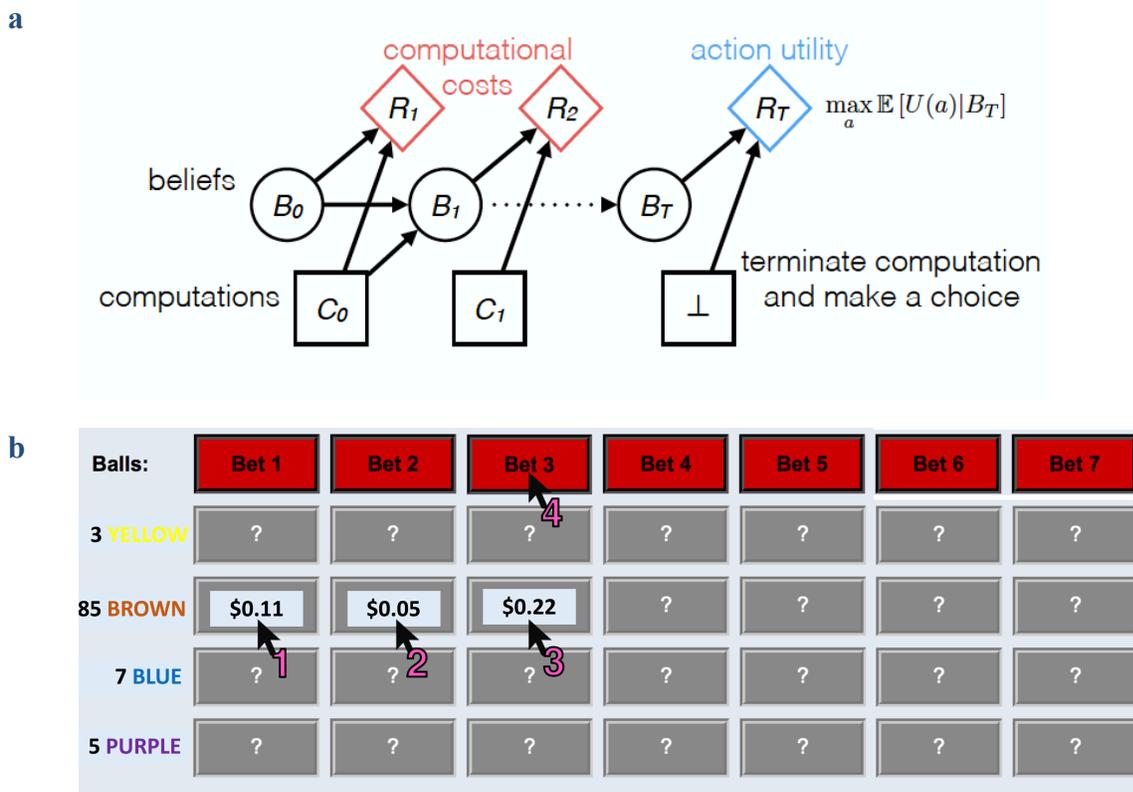


Figure 6: Automatic strategy discovery applied to multi-alternative risky choice in the MouseLab paradigm.

a: Illustration of a meta-level Markov decision process.

b: Illustration of the automatically discovered SAT-TTB heuristic. To choose between the alternatives (e.g., Bet 1 to Bet 7), SAT-TTB looks at their payoffs for the most probable outcome (e.g., a brown ball being drawn from the urn) until it encounters a payoff that meets or exceeds its aspiration level (e.g., \$0.16). As soon as this happens, SAT-TTB stops collecting information and chooses the current alternative. The question marks mark payoffs that the SAT-TTB heuristic did not look at. The SAT-TTB heuristic with an aspiration level of about \$0.16 is optimal in this environment because processing a payoff costs the equivalent of \$0.01, the highest possible payoff is \$0.25, and the most likely outcome is much more likely than the second most likely outcome.

Chapter 7: Conclusions of Part 1

Overall, the findings summarized above suggest that, contrary to the bleak picture painted by previous research on heuristics and biases, human cognition is not fundamentally irrational and can be understood as making rational use of bounded cognitive resources. By reconciling rationality with cognitive biases and bounded resources, this line of research addresses fundamental problems of previous rational modeling frameworks, such as expected utility theory, logic, and probability theory. Resource rationality might thus come to replace classical notions of rationality as a theoretical foundation for modeling human judgment and decision-making in economics, psychology, neuroscience, and other cognitive and social sciences.

Part 2: Expanding the bounds on human rationality

In the second part of my dissertation, I apply the principle of resource rationality to develop tools and interventions for improving the human mind. Since human rationality depends on the fit between the mind and the structure of the environment, there are two complementary approaches to rationality enhancement. The first approach is to restructure the environment so people's heuristics perform better; the second is to discover heuristics that are well adapted to the structure of the environment and teach them to people. I pursued each of these approaches in turn.

Chapter 8: A Cognitive Prosthesis for Goal Achievement

The success of resource rational analysis seems to suggest that we can usually find an environment for which people's heuristics are resource rational. That environment is not always identical to the one in which people find themselves, and this mismatch may be responsible for many of the systematic errors that plague human decision-making. This perspective suggests that we can help people overcome these biases by redesigning the way that alternative courses of action are presented to account for people's heuristics. In my dissertation, I instantiated this general approach into a computational method for repairing broken incentive structures and a to-do list gamification app that helps people overcome procrastination (see Figure 7a; Lieder, Chen, Krueger, & Griffiths, 2019).

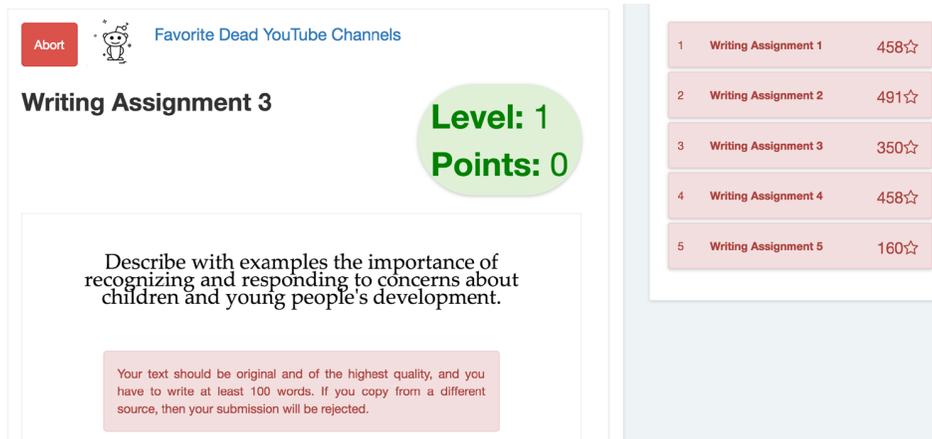
Procrastination is often a consequence of people's tendency to avoid or perform actions according to their immediate costs and benefits (Steel, 2007). This admittedly short-sighted strategy works well when immediate rewards are aligned with long-term benefits. Optimal gamification (Lieder, Chen, Krueger, & Griffiths, 2019) therefore aligns each action's immediate reward with its long-term benefits by adding incentives in the form of game elements, such as points, levels, and badges. I proved that even the most short-sighted heuristics lead to optimal decisions when taking action a in state s is incentivized by

$$PR(s, a) = \mathbb{E}[V^*(S_{t+1})|s_t, a] - V^*(s), \quad (3)$$

where V^* is the optimal value function of a Markov decision process model of the decision environment (Sutton & Barto, 2018). After a series of controlled experiments in a simple game environment showed that this approach was effective (Lieder & Griffiths, 2016), I applied it to develop a to-do list app that uses optimal gamification to help people overcome procrastination and take action to achieve their goals on time (Lieder, Chen, Krueger, & Griffiths, 2019). As shown in Figure 7, optimal gamification significantly increased the proportion of people who started their writing assignments on time to finish them before the deadline from less than 60%

to more than 85% ($\chi^2(1) = 11.20, p < 0.001, w = 3.35$, 95% confidence intervals: [67.6%, 91.5%] without optimal gamification, [75.7%, 93.9%] with optimal gamification). This was not an effect of gamification per se because adding the same number of points to all tasks was ineffective (see Figure 7b).

a



b

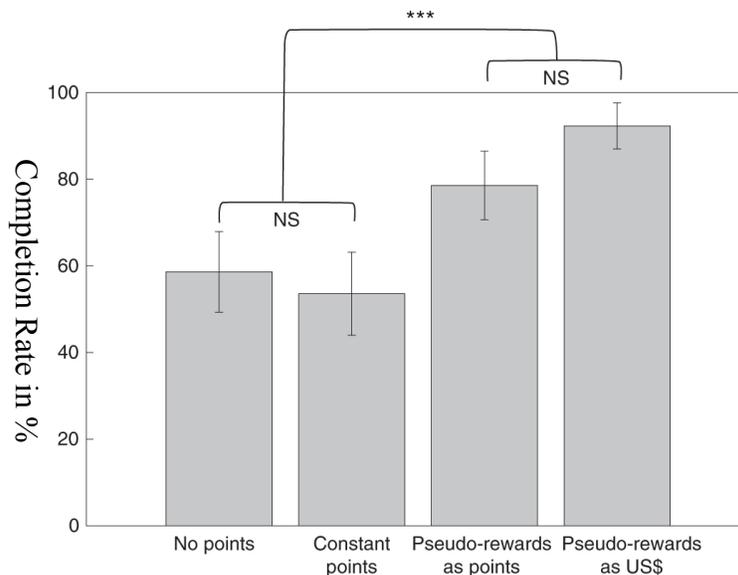


Figure 7: Optimal gamification reduces procrastination. A: Screenshot from the experimental condition with optimal gamification. B: Proportion of participants who completed all assignments on time (error bars show ± 1 s.e.m.).

Chapter 9: An Intelligent System that Teaches People Optimal Cognitive Strategies

Early interventions for improving human rationality educated people about cognitive biases and taught them the normative principles of logic, probability theory, and expected utility theory. The practical benefits of such interventions are limited because the computational demands of applying them to the complex problems people face in everyday life far exceed individuals' cognitive capacities. Instead, the principle of resource rationality suggests that people should rely

on simple, computationally efficient heuristics that are well-adapted to the structures of their environments. Building on this idea, I leveraged the automatic strategy discovery method and insights into metacognitive learning from the first part of my dissertation to develop intelligent systems that teach people resource rational cognitive strategies. I illustrate this approach by developing and evaluating a cognitive tutor that trains people to plan resource rationally. My results show that practicing with the cognitive tutor improves people's planning strategies significantly more than practicing without feedback. Follow-up experiments demonstrate that this training effect transfers to more difficult planning problems in novel and more complex environments and that this transfer effect is retained over time. This indicates that discovering and teaching resource rational heuristics may be a promising approach to improving human judgment and decision-making (Lieder, Callaway, et al., 2017, 2019, in revision).

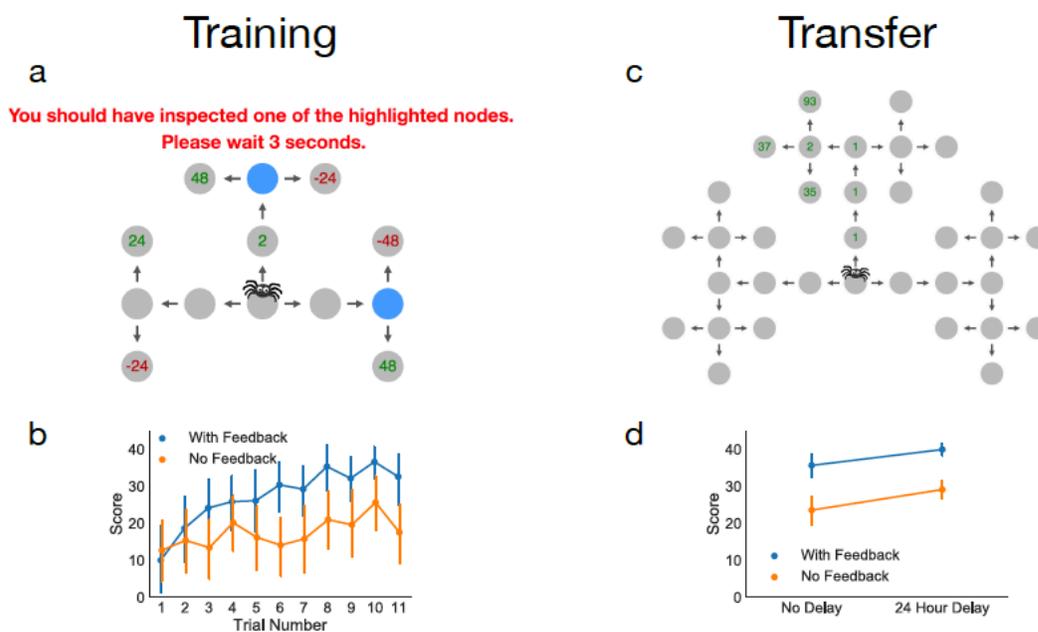


Figure 8: (a) Example feedback from the cognitive tutor in the training phase. (b) Participants learn to achieve high scores faster with the tutor's feedback. (c) A more difficult transfer problem: Feedback is not given in either condition. (d) Participants who received feedback in the training phase outperformed control participants when tested immediately or after a 24-hour delay.

General Conclusion

Through integrating rational principles with cognitive constraints, resource rationality provides a realistic normative standard for human reasoning and decision-making. My findings about human rationality and metacognitive learning are consistent with the view that evolution and learning adapt the mind to the structure of its environment and the constraints imposed by its limited resources. These adaptive mechanisms appear to optimize for resource rationality, and the benefits of training with the cognitive tutor demonstrate that this adaptation can be accelerated with the help of artificial intelligence. This makes resource rationality a promising theoretical framework for modeling and improving human cognition.

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